Linking Powerful Mathematical Ideas and Developmental Learning Outcomes in Early Childhood Mathematics

Bob Perry
University of Western Sydney
b.perry@uws.edu.au

Sue Dockett
University of Western Sydney
s.dockett@uws.edu.au

Elspeth Harley
South Australian Department of Education and Children's Services
harley.elspeth@saugov.sa.gov.au

Nicole Hentschke
Enfield Kindergarten
kindy.director@enfieldkgn.sa.edu.au

In recent years, there has been a marked increase in the expectations of the mathematical performance of Australian children in their first year of school. Partly, this has been the result of Australasian research emanating from large systemic early childhood numeracy projects. One result of this increase in expectations for the first year of school has been a subsequent change in the expectations held by teachers and parents of children's mathematical learning in prior-to-school settings and a consequent formalisation of mathematical teaching practices in many of these settings. This paper reports work done with preschool educators as part of the Southern Numeracy Initiative in South Australia in which the educators sought an alternative to the push for further formalisation. The results include the identification of 'powerful ideas' in mathematics, the linking of these to the Developmental Learning Outcomes in the mandated South Australian curriculum documents through pedagogical enquiry questions within a numeracy matrix and how learning stories (narrative assessment) could be used to provide an assessment regime capable of celebrating mathematical learning while remaining compatible with key principles of preschool education. In particular, this paper considers the development and use of the numeracy matrix.

Over the last decade, mathematics education in the first years of Australasian primary schools has been characterised by systemic programs such as *Count Me In Too* (Bobis & Gould, 2000), *Early Numeracy Research Program* (Clarke & Clarke, 2004), *First Steps* (Willis, Devlin, Jacob, Treacy, Tomazos, & Powell, 2004), and *Numeracy Development Program* (Thomas, Tagg, & Ward, 2003). In each of these programs, evidence has been gathered indicating that many young children are capable of much higher level mathematics than had previously been expected of them in the first years of school. "There is considerable research to show that many children arrive at primary school already equipped with an extraordinary understanding of the number system and how it works" (Bobis et al., 2005, p. 27). This research has been reflected in syllabus documents (see, for example, Board of Studies, NSW, 2002; Department of Education, Training and Employment, 2001) with consequent pressure on teachers in the first years of school to raise their expectations. Since they are now being asked to bring children's learning to this higher standard, these teachers have taken renewed interest in the mathematical capabilities of children as they start school.

Thus, there is some pressure being exerted on prior-to-school educators and parents to ensure, wherever possible, that children starting school can perform at a higher mathematical level than previously expected and that they are able to succeed at more formal mathematics than had previously been the case (Perry & Dockett, 2005a). There is often conflict between this increase in formality and the play-based, child-centred philosophies of prior-to-school settings (Thomson, Rowe, Underwood, & Peck, 2005).

The Southern Numeracy Initiative

The Southern Numeracy Initiative (SNI) was established in 2004 among five high schools, sixteen primary schools and six preschools in two districts south of Adelaide. The aims of SNI included the following:

- to develop and implement successful teaching and learning practices to improve numeracy; and
- to challenge teachers to explore their beliefs and understandings about how children develop their understanding of mathematics, and how this can be supported through the teaching program.

The preschools involved in SNI had some concerns about the direction being taken by the schools involved, especially in terms of apparent tension between the formality of instruction and the methods of assessment chosen by the schools and the child-centred, play-based approaches that characterised their early childhood programs. As a consequence, two of the authors of this paper were invited to work with the preschool educators in SNI to develop a program aimed at improving teaching, learning and assessment practices in the numeracy development of young children. The key research question for the overall project was: *how can the powerful mathematical ideas that are displayed by young children before they start school be recognised and celebrated in a valid manner within the context of a mandated reporting regime and a child-centred, play-based approach to learning.* This paper reports one outcome from this work.

Powerful Mathematical Ideas

Preschool educators tend, at least in Australia, to reject the divided, content-based approach to mathematics curriculum which is often used in schools (Doig, McCrae, & Rowe, 2003). There is, however, general agreement that all children in their early childhood years are capable of accessing powerful mathematical ideas that are both relevant to their current lives and form a critical foundation for their future mathematical learning and that children should be given the opportunity to access these ideas through high quality child-centred activities in their homes, communities, and prior-to-school settings (Kilpatrick, Swafford, & Findell, 2001; Perry & Dockett, 2005a; Thomson et al., 2005). What are these ideas?

Two of the authors of this paper have constructed a list of powerful mathematical ideas that they have used for some time to plan, observe, facilitate and assess young children's mathematical learning (Perry & Dockett, 2002, 2005b). Their list bears many similarities to other such lists (see, for example, Greenes, Ginsburg, & Balfanz, 2004; National Council of Teachers of Mathematics, 2000) but was felt to be most appropriate to the SNI project. The powerful mathematical ideas are:

- Mathematisation;
- Connections;
- Argumentation;
- Number sense and mental computation;
- Algebraic reasoning;
- Spatial and geometric reasoning; and
- Data and probability sense.

These powerful mathematical ideas are defined in Perry and Dockett (2002).

Developmental Learning Outcomes

The South Australian Department of Education and Children's Services is responsible for the education of children in preschools, primary and secondary schools throughout the state. A key curriculum document across this broad span is *The South Australian Curriculum, Standards and Accountability (SACSA) Framework* (Department of Education, Training and Employment, 2001). Educators in South Australian preschools and schools are accountable to this framework. In the preschool year, this accountability for children's learning is assessed against eight Developmental Learning Outcomes (DLOs)—broad, observable and assessable consequences of the curriculum that reflect the integration of learning and development and allow for the different developmental pathways of individual children. The Developmental Learning Outcomes are:

- Children develop trust and confidence;
- Children develop a positive sense of self and a confident personal and group identity;
- Children develop a sense of being connected with others and their world;
- Children are intellectually inquisitive;
- Children develop a range of thinking skills;
- Children are effective communicators;
- Children demonstrate a sense of physical wellbeing; and
- Children develop a range of physical competencies.

This paper reports how the powerful mathematical ideas and the developmental learning outcomes were brought together by a group of practising early childhood educators into a numeracy matrix which encouraged the educators to plan, implement and assess their practices against the developmental learning outcomes and the powerful mathematical ideas displayed by their children.

Constructing the Numeracy Matrix

Two of the authors of this paper worked with a small group of early childhood educators for two days in 2005 and another day in early 2006. On the first day, background information was presented and discussed on the nature of powerful mathematical ideas and their relevance to early childhood. Participants agreed to use the powerful mathematical ideas presented in their planning and assessment of children's learning outcomes. As well, participants were introduced to the *learning stories* (narrative assessment) methodology for assessment (Carr, 2001) and were invited to use this methodology in their settings. The second professional development day emphasised links between the developmental learning outcomes from the *SACSA Framework* and the powerful mathematical ideas introduced on the first day. During the second day, participants were introduced to the notion of the numeracy matrix and provided with some exemplary cells in the matrix. Part of their task on this day was to complete other cells in the matrix. The third professional development day was held in March, 2006 and allowed the early childhood educators to share their experiences with the numeracy matrix and with the learning stories assessment approach. Further meetings are planned during 2006 to continue the refinement of the matrix and the development of the educators' expertise in using it in their settings.

The Numeracy Matrix

The theoretical basis for the numeracy matrix is that the key determinants of children's successful outcomes are the pedagogical relationships and practices of educators (Laevers & Heylen, 2004). Hence, the elements of the matrix that bring the DLOs and the powerful mathematical ideas together are 'pedagogical questions'—questions asked of early childhood educators as to what practices they are using to ensure that their children's learning outcomes for both the powerful mathematical ideas and the DLOs are developing. The numeracy matrix constructed during the SNI professional development days consists of 56 cells (eight DLOs x seven powerful mathematical ideas) in which each cell provides examples of pedagogical questions early childhood educators can ask themselves as they teach towards, assess or report on the DLOs while, at the same time, take cognisance of the appropriate mathematical development of their children.

The term 'numeracy matrix' is used here for two reasons. Firstly, it has been argued elsewhere that in the early childhood years, the nature and content of mathematics and numeracy learning are so similar that there is not a lot to be gained by trying to make clear distinctions between the two areas. Secondly, the title is a reflection of the title of the project from which it is derived: *The Southern Numeracy Initiative*.

An example of one of the cells of the numeracy matrix is presented in Table 1.

Table 2: An example of a cell from the numeracy matrix

Powerful mathematical idea	DLO: Children develop a range of thinking skills
Data and Probability Sense	How do we encourage children to develop a notion of fairness in their lives?
	In what ways do we provide opportunities for children to monitor change over time?

In this cell are two pedagogical questions which challenge early childhood educators to inquire as to what they are doing to help children develop the mathematical idea and the developmental learning outcome. The answers to these questions will affirm those educators who are working towards these goals, as well as suggest to them that more activities might be needed to help the children develop further. The questions will also stimulate educators who have not considered their practices in these areas to investigate the relevance of current activities and practices or the need for new practices. Obviously, these pedagogical questions have relevance to other key learning areas as well as mathematics, thus emphasising integration of mathematics learning with other learning areas.

One of the ways in which the SNI early childhood educators chose to conceptualise the matrix was in terms of the level of outcomes it encompassed. Using Harley's (2005) model, the developmental learning outcomes could be seen as the 'castle' for which the educators are striving. That is, they are the transformational goals for children's learning consisting of integrated skills and understandings. In our context, the powerful mathematical ideas can be characterised as the 'walls' from which, at least in part, the castles are built. These are the transitional goals for children's learning consisting of higher order competencies and processes. In castle constructions, walls are built from 'bricks' and in our context, the early childhood educators saw the bricks as the traditional goals for children's mathematical learning, consisting of the small segments of learning achieved through tasks structured by educators. In some ways, these bricks were the practices and activities identified by the

pedagogical questions contained in the body of the matrix. As one SNI early childhood educator suggested:

> The matrix encompasses all the levels that I need to consider when I am planning and assessing children's mathematical development. I know that the bricks are important but in the past, that was about as far as I got. Now I can see that I need to think about the walls and the overall castle whenever I do something. I am trying to build castles and everything I do has to be aimed at doing that. Most of it is but I had never thought about it like that before.

It is not possible in the space available for this paper to present the entire numeracy matrix. Hence, only two segments will be provided. The first, in Table 2, provides the complete description of one powerful mathematical idea against the eight cells representing each of the DLOs. In this way, it can be seen how the 'bricks' (the pedagogical questions and the practices and activities derived from them) relate to one of the 'walls' (the powerful mathematical idea). Table 3, on the other hand, provides the complete description of one DLO against all of the powerful mathematical ideas, thus relating a 'castle' to its 'walls'.

Table 2: *Numeracy matrix for powerful mathematical idea: Argumentation*

Developmental Learning Outcomes	Powerful mathematical idea: Argumentation
Children develop trust and confidence	What opportunities and support do we give children to explore and take risks as they justify their mathematical thinking? How do we encourage children to demonstrate flexibility and to manage different mathematical ideas as they are presented to them by peers?
Children develop a positive sense of self and a confident and personal group identity	How do we encourage children to develop and maintain respectful relationships with adults and children even though they may not agree with their mathematical ideas? How do we encourage children to help develop agreed values and sociomathematical norms of behaviour in their groups?
Children develop a sense of being connected with others and their worlds	How do we encourage children to contribute constructively to mathematical discussions and arguments? How do we encourage children to question why their and other people's mathematical ideas work? What opportunities do we provide for children to communicate their own mathematical ideas to a respectful group of peers?
Children are intellectually inquisitive	What opportunities do we give children to put forward a mathematical argument and to justify it? How do we assist children to gain confidence in their ability to explore, hypothesise and make appropriate choices in their mathematics?
Children develop a range of thinking skills	How do we encourage children to participate in group discussion and justification about the solution of mathematical problems? What opportunities do we provide for children to suggest alternative solutions to mathematical problems?
Children are effective communicators	How do we encourage children to interact with others to explore ideas, negotiate possible solutions and share their mathematical learning? What opportunities do we provide for each child to use different communication strategies to help clarify their and their peers' mathematical thinking?
Children develop a sense of physical well being	What opportunities do we provide for children to develop confidence in expressing their mathematical ideas? How do we encourage children to celebrate their efforts and achievements in mathematics learning?

The powerful mathematical idea of argumentation:

> allows children … to justify not only their own mathematical thinking but also to distinguish between the strengths of arguments and whether the mathematics being constructed within the arguments is actually different from previous mathematical arguments that have been interactively constructed (Perry & Dockett, 2002, p. 92).

Providing such justification, while clearly important as children develop their mathematics, is also important in many other areas of learning and certainly contributes in numerous ways to the developmental learning outcomes. On the other hand, the development this powerful mathematical idea depends on early childhood educators' pedagogical practices, some of which are presented in the form of questions or challenges within the numeracy matrix. For example, in answer to the pedagogical question "How do we encourage children to contribute constructively to mathematical discussions and arguments?" one of the SNI early childhood educators suggested:

> We would firstly need to make sure that children felt safe in talking up about their solutions and those of others. We want them to say what they think but in ways that will not hurt anyone. That will depend a lot on the atmosphere in the group but it will also need the kids to know the maths that they are talking about.

Table 3: Numeracy matrix for the DLO: Children are intellectually inquisitive

Powerful mathematical idea	DLO: Children are intellectually inquisitive
Mathematisation	What opportunities do we give children to experiment with mathematical concepts and representations in problem solving and investigation? How do we encourage children to gather information and ask questions that might be answered by this information?
Connections	What opportunities do we give children to investigate mathematical ideas that are part of the local natural and constructed environment? How do we encourage children to use mathematics to be a critical consumer of everyday products? How do we assist children to find connections between different mathematical concepts and representations?
Argumentation	What opportunities do we give children to put forward a mathematical argument and to justify it? How do we assist children to gain confidence in their ability to explore, hypothesise and make appropriate choices in their mathematics?
Number sense and mental computation	What opportunities do we give children to explore, hypothesise, take risks and engage in symbolic and dramatic play with confidence?
Algebraic reasoning	What opportunities do we give children to experiment with word, language, number and shape patterns? How do we encourage children to explore patterns using their senses? How do we assist children to use pattern making and pattern continuation for problem solving and investigation?
Spatial and geometric thinking	What opportunities do we give children to explore their local environment and record what they see using visual means? How do we encourage children to analyse critically the shapes found on the supermarket shelves? How do we assist children to compare and classify shapes?
Data and probability sense	What opportunities do we give children to investigate different forms of data representation? How do we encourage children to interpret data arising from the use of everyday products? How do we assist children to gather information, ask questions, seek clarification and consider possibilities about their own lives?

The contribution of each powerful mathematical idea to the development learning outcome is clearly illustrated through this part of the overall matrix. Very few of the early childhood educators involved in SNI would have argued against mathematics contributing to the DLO of children being intellectually inquisitive. However, very few of these educators were able to articulate how that might occur in a learning area such as mathematics with its perceived underlying (and constraining) structure. Through their use of the numeracy matrix, the educators are now able to see how each of the powerful ideas contributes to the DLO. One of them was able to suggest that the work with the numeracy matrix had helped them see how the DLOs were the capstones to all that they were trying to do in all learning areas.

> When I thought about shapes and geometry, I thought all that was needed was for the children to know the names of some regular shapes. It was really not something I thought they would be inquisitive about. By using the matrix, I can see that they can develop their inquisitiveness by asking lots of questions about lots of different shapes in their environment — not just triangles and circles — and can investigate why things are the way they are. This will take them into asking about how things are used, where they come from, whether some shapes are better than others for a particular job and why some shapes look better than others. It is exciting for the children — and for me!

Conclusion

The purpose of this paper was to introduce the numeracy matrix which has been developed as part of the Southern Numeracy Initiative in South Australia and to celebrate the work of the early childhood educators who have been involved in its development. This is an early paper in the continuing SNI project but it does illustrate the power of collaborative effort between researchers and practitioners and how each can become the other in the course of the project. Already, there is evidence that the use of the numeracy matrix and the thinking behind it have had positive effects on the pedagogical practices of the early childhood educators involved. This, coupled with the learning stories assessment methodologies that will be the subject of subsequent papers and presentations, seems to suggest that the SNI preschool project will lead to improved practices and, consequently, improved learning outcomes for the children who are fortunate enough to be taught by this enthusiastic group of educators. In the words of one of the educators:

> The numeracy matrix has helped me to rethink the way I am teaching and the way children are learning at my centre. It is a useful document in the planning and evaluation not only of children's learning but in your own teaching pedagogy. It poses questions that look not just at the skills that children are required to have in numeracy but it looks deeper into 'how' and 'what' we do to encourage and provide opportunities for children to experience and develop mathematical understandings.

References

Board of Studies, NSW. (2002). *Mathematics K-6 syllabus.* Sydney: Author. Available online from http://www.boardofstudiesnsw.edu.au

Bobis, J., Clarke, B., Clarke, D., Thomas, G., Wright, R., Young-Loveridge, J., & Gould, P. (2005). Supporting teachers in the development of young children's mathematical thinking: Three large scale cases. *Mathematics Education Research Journal,* 16(3), 27-57.

Bobis, J., & Gould, P. (2000). Changing the professional knowledge of teachers. In J. Bana, & A. Chapman, (Eds.), *Mathematics education beyond 2000* (Proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia, Fremantle, pp. 47-54). Sydney: MERGA.

Carr, M. (2001). *Assessment in early childhood settings: Learning stories*. London: Paul Chapman.

Clarke, D. M., & Clarke, B. A. (2004). Mathematics teaching in Grades K-2: Painting a picture of challenging, supportive, and effective classrooms. In R. N. Rubenstein & G. W. Bright (Eds.), *Perspectives on the teaching of mathematics* (66th Yearbook of the National Council of Teachers of Mathematics, pp. 67-81). Reston: NCTM.

Department of Education, Training and Employment. (2001). *South Australian curriculum, standards and accountability framework*. Adelaide: Author. Available online from: http://www.sacsa.sa.edu.au

Doig, B., McCrae, B., & Rowe, K. (2003). *A good start to numeracy: Effective numeracy strategies from research and practice in early childhood.* Melbourne: Australian Council for Educational Research. Available online from: http://www.dest.gov.au/schools/publications/2003/index.htm

Greenes, C., Ginsburg, H. P., & Balfanz, R. (2004). Big math for little kids. *Early Childhood Research Quarterly,* 19(1), 159-166.

Harley, E. (2005). *Curriculum accountability framework for preschools: Supporting, assessing and reporting for children's learning and development.* Unpublished report, South Australian Department of Education and Children's Services.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Laevers, F., & Heylen, L. (Eds.). (2004). *Involvement of children and teacher style: Insights from an international study on experimental education.* Leuven, Belgium: Leuven University Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Perry, B., & Dockett, S. (2002). Young children's access to powerful mathematical ideas. In L. D. English (Ed.). *Handbook of international research in mathematics education: Directions for the 21st century* (pp. 81-111). Mahwah, NJ: Lawrence Erlbaum.

Perry, B., & Dockett, S. (2005a). "I know that you don't have to work hard": Mathematics learning in the first year of primary school. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th conference of the International Group for the Psychology of Mathematics Education,* (Vol. 4, pp. 65-72). Melbourne: University of Melbourne.

Perry, B., & Dockett, S. (2005b). What did you do in maths today? *Australian Journal of Early Childhood. 30*(3), 32-36.

Thomas, G., Tagg, A., & Ward, J. (2003). *An evaluation of the Early Numeracy Project 2002: Exploring issues in mathematics education.* Wellington: Ministry of Education.

Thomson, S., Rowe, K., Underwood, C., & Peck, R. (2005). *Numeracy in the early years: Project Good Start.* Melbourne: Australian Council for Educational Research.

Willis, S., Devlin, W., Jacob, L., Treacy, K., Tomazos, D., & Powell, B. (2004). *First steps in mathematics*. Sydney: Rigby Heinemann.